

2-State Asymmetries : Matter-Antimatter and Other Effects

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Abstract

This paper is based on the framework of a two dimensional universe considering it to be a lattice structure. The *Ising model* has been used to show that there is a spontaneous phase transition from the *Planck scale* to the *Compton scale*. Also, a mean field theoretic approach has been considered to delve into the statistical properties of such a lattice-structured universe. All this answers questions like matter-antimatter asymmetry.

1 Introduction

The *Planck scale* is considered to be a fundamental aspect of the universe. In terms of size, the Planck scale is extremely small and thus the phenomena encompassing it are obscure. Currently, the Planck length ($\sim 10^{-33}cm$) is regarded as the fundamental minimum length. Several authors have worked with the *Planck scale* in their papers [1, 2, 3, 4, 5, 6]. In contradistinction, the author Sidharth had considered the *Compton scale* (where, the Compton length is rudimentary) and derived the accelerating nature of the universe [7]. Also, Sidharth has shown that there is a transition [8, 9] from the *Planck scale* to the *Compton scale* by dint of the *Landau-Ginzburg* phase transition.

Besides, other authors have also considered such phase transitions from the *Planck scale* to the *Electroweak scale* [10, 11] (and several references therein) where the *Compton length* ($l = \frac{\hbar}{mc}$) should serve as a key feature since the particles acquire mass via the *Higgs mechanism*. We resort to the terminology *Compton scale* in lieu of *Electroweak scale*, in this

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paper.

The current work is based on the *Ising model* of phase transitions in ferromagnetic materials. In the first section, we consider a two dimensional universe and follow Peierls' methodology [12, 13] to substantiate that there can indeed be a transition from the *Planck scale* to the *Compton scale*. In the second section, we consider a mean field theoretic approach and try to find the statistical properties of the system interacted upon by the *zpf*. In the third section we show that the effects of the *Planck scale* will be negligible and the *Compton scale* will be prevalent in the long run. In the final section, we discuss about some other 2-state cases where there is an asymmetry between the states and endeavour to explain them in light of this model.

2 An analogue of the Ising Model

We commence by taking into account a 2D universe and consider it as a lattice structure comprised of particles which are termed as lattice sites. There is a set of lattice sites S , each having a set of adjacent sites forming the 2D lattice structure. For each lattice site $s \in S$ there is a discrete variable ϕ_s such that $\phi_s \in \phi_c, \phi_p$, where ϕ_c and ϕ_p represent the states corresponding to the *Compton scale* and the *Planck scale* respectively. Precisely, ϕ_p describes the properties of the *Planck scale* or the *Planck phase* and ϕ_c describes the properties of the *Compton scale* or the *Compton phase* of a lattice site. This particular configuration, $\phi = \phi_s$ is an assignment of state to each lattice site $s \in S$.

Now, for any two adjacent sites $s_1, s_2 \in S$ we have the energy due to nearest neighbour interaction as $E_{(s_1, s_2)}$. Here, we consider that any site $s \in S$ is interacted upon by the external *zero point field* (*zpf*) giving the corresponding energy F_s . Before the intrusion of the *zpf* the sites have the state ϕ_p , i.e the entire universe is in the *Planck phase*. The *zpf* influences the sites to take up the configuration ϕ_c . So, for the state configuration we have the Hamiltonian

$$H(\phi) = - \sum_{\langle s_1, s_2 \rangle} E \phi_{s_1} \phi_{s_2} - \sum_s F \phi_s \quad (1)$$

where, we have simply written $E_{(s_1, s_2)} = E$ and $F_s = F$. The first sum is over all pairs of nearest neighbours in the lattice structure and the second sum has been taken over all lattice sites. Before the *zpf* field comes into play all sites are in the *Planck state* (ϕ_p).

Now, as an analogue to the *Ising model* [13], we consider here that in presence of the *zpf* the lattice sites conform with the configuration ϕ_c and when the *zpf* is absent they exhibit the behaviour of spontaneously taking up the configuration ϕ_c , in analogy with the the ferromagnetic case. As, the number of lattice sites n tends to infinity the boundary of the lattice structure moves away to infinity. Considering the *zpf* to be acting on the boundary one can relate this as the absence of the *zpf*.

We will try to find the probability of a site s_0 having the configuration $\phi_0 = \phi_p$, which is at the deep central region of the lattice structure. In our case also, we consider that the effect of the *zpf* in the boundary region will flow towards all lattice sites at low temperatures.

Albeit, at high temperatures this effect will be compensated by the random fluctuational nature of the sites, on account of high kinetic energy.

Now, in the ferromagnetic case [12] the *up* state is written as (+1) and the *down* state as (-1) and thereby we have the usual multiplication rules. Here, in analogy we begin by making the assumption that

$$\phi_c \phi_p = \phi_p$$

i.e., the interaction of two neighbour sites corresponding to two different states ϕ_c {analogously (+1) state of the ferromagnetic case} and ϕ_p {analogously (-1) state of the ferromagnetic case} reproduce the state describing the *Planck scale*. Also, we presume that the interaction of the two neighbour sites corresponding to the same states will produce the state ϕ_c corresponding to the *Compton scale*. Now, the probability for a configuration ϕ is given as

$$P(\phi) = \frac{\exp\{-\beta H(\phi)\}}{Z} \quad (2)$$

where, Z is the *partition function* given by the relation

$$Z = \sum_{\phi \in \Phi} \exp\{-\beta H(\phi)\} \quad (3)$$

Here, Φ is the set of all possible configurations in the entire lattice structure. Now, since ϕ_0 corresponds to the lattice site s_0 at the central region, we have [13]

$$P(\phi_0 = \phi_p) = \frac{1}{Z} \sum_{\phi \in \Phi_0} \exp\{-\beta H(\phi)\} \quad (4)$$

where, Φ_0 corresponds to the set of all configurations ϕ for which $\phi_0 = \phi_p$ and $\Phi_0 \subseteq \Phi$ since Φ_0 may be equal to Φ or just it's subset. Now, since the absence of the *zpf* means that it acts on the boundary we consider a situation analogous to the *Ising model* [13]. The lattice sites with $\phi_s = \phi_p$ are *lands* in the *ocean* of the sites with $\phi_s = \phi_c$. Considering a *shoreline* or *boundary* to be a close path connecting the midpoints of adjacent square lattices we see that it merely separates ϕ_c in the *ocean* from ϕ_p in the *land*. Consequently, a *shoreline* or *boundary* accounts for the set of juxtapositions $\langle s_1, s_2 \rangle$ for which we have: $\phi_{s_1} \phi_{s_2} = \phi_p$. Now, if a *shoreline* L of length l ($= l' \phi_p$, where $\sum_{\langle s_1, s_2 \in L \rangle} \phi_{s_1} \phi_{s_2} = l' \phi_p$) is drawn around the site s_0 then the probability of the set of configurations $\Phi_L \in \Phi_0$ having L as a *shoreline* is given by

$$P(\Phi_L) = \frac{1}{Z} \sum_{\phi \in \Phi_L} \exp\{-\beta H(\phi)\} = \frac{1}{Z} \exp\{-\beta E l\} \sum_{\phi \in \Phi_L} \exp\{\beta E \sum_{\langle s_1, s_2 \notin L \rangle} \phi_{s_1}, \phi_{s_2}\} \quad (5)$$

Again, we can have another configuration ϕ' by changing the states inside the shoreline L , where we have $\phi \in \Phi_L$. Since, for a fixed *shoreline* L the mapping

$$M : \phi \mapsto \phi'$$

is injective, we can say that Φ'_L is the image of Φ_L under M . Thus, we have

$$\sum_{\langle s_1, s_2 \notin L \rangle} \phi_{s_1} \phi_{s_2} = \sum_{\langle s_1, s_2 \rangle} \phi'_{s_1} \phi'_{s_2} - l$$

i.e., we have

$$\sum_{\langle s_1, s_2 \notin L \rangle} \phi_{s_1} \phi_{s_2} < \sum_{\langle s_1, s_2 \rangle} \phi'_{s_1} \phi'_{s_2}$$

Therefore, we have from equation (5)

$$P(\Phi_L) < \frac{1}{Z} \exp\{-\beta El\} \sum_{\phi \in \Phi'_L} \exp\{\beta E \sum_{\langle s_1, s_2 \notin L \rangle} \phi'_{s_1} \phi'_{s_2}\}$$

Or,

$$P(\Phi_L) < \frac{1}{Z} \exp\{-\beta El\} \sum_{\phi \in \Phi'_L} \exp\{-\beta \sum_{\langle s_1, s_2 \notin L \rangle} H(\phi')\}$$

Since the mapping M is injective the sum over Φ_L has been replaced by Φ'_L and now we replace Φ_L by Φ since $\exp\{-\beta H(\phi)\} > 0$. So, we get

$$P(L) < \exp\{-\beta El\} \frac{1}{Z} \sum_{\phi \in \Phi} \exp\{-\beta H(\phi)\}$$

Again, since

$$Z = \sum_{\phi \in \Phi} \exp\{-\beta H(\phi)\}$$

we have

$$P(L) < \exp\{-\beta El\} \tag{6}$$

Now, this was for one shoreline L . Considering all *shorelines* belonging to the set $\{L\}$ surrounding the lattice site s_0 we have

$$P(\phi_0 = \phi_p) = \sum_{L \in \{L\}} P(L)$$

which means

$$P(\phi_0 = \phi_p) < \sum_{L \in \{L\}} \exp\{-\beta E l\} = \sum_{l=4}^{\infty} L(l) \exp\{-\beta E l\} \quad (7)$$

where, $L(l)$ denotes the number of *shorelines* surrounding the lattice site s_0 and a succession of length l is comprised of l *independent steps* [12]. Now, since the *shorelines* are the line segments nearest to s_0 that connect the midpoints of the adjacent square lattices we have for each *shoreline* counted l times as *independent steps*

$$l \times L(l) < \frac{l^2}{2} 4^l$$

since the *independent steps* have $\frac{l^2}{2}$ starting points and since there are four choices for four nearest neighbours (in 2D). Therefore, we have

$$P(\phi_0 = \phi_p) < \frac{1}{2} \sum_{l=4}^{\infty} l (4 \exp\{-\beta E\})^l$$

Or, more precisely we may write

$$P(\phi_0 = \phi_p) < \frac{1}{2} \sum_{l=1}^{\infty} l (4 \exp\{-\beta E\})^l \quad (8)$$

Now, writing $x = 4 \exp\{-\beta E\}$ ($x < 1$), we have the following series relation [13]

$$\frac{x}{1-x} = \sum_{l=1}^{\infty} l x^l$$

which comes by differentiating the relation

$$\frac{1}{1-x} = \sum_{l=1}^{\infty} x^l$$

and then multiplying it by x . Thus, we have finally the relation

$$P(\phi_0 = \phi_p) < \frac{1}{2} \left[\frac{4 \exp\{-\beta E\}}{1 - 4 \exp\{-\beta E\}} \right] \quad (9)$$

Now, it is obvious from the above that for large β and consequently low temperature T we have the probability

$$P(\phi_0 = \phi_p) < \frac{1}{2}$$

independent of the lattice structure and size. Also, if the neighbour-interaction energy E

is very high $P(\phi_0 = \phi_p)$ becomes considerably small. Thus, the probability for the site to be in the state ϕ_c is

$$P(\phi_0 = \phi_c) = 1 - P(\phi_0 = \phi_p)$$

When, $P(\phi_0 = \phi_p)$ is very small for large β and E we have $P(\phi_0 = \phi_c)$ quite large. This means ostensibly that in case of low temperatures and high neighbour-interaction energies there is a *phase transition* from the *Planck scale* to the *Compton scale* and universe spontaneously undergoes this transition on account of the *zero point field*. Precisely, we can infer that there exists a critical temperature $T = T_c$ at which this phase transition occurs spontaneously.

Inherently, for $P(\phi_0 = \phi_p)$ very small we shall find in the subsequent sections that the *Compton scale* becomes prevalent and the effects of the *Planck scale* fades away.

3 Mean Field Theoretic Approach

In this section we resort to the *mean field theory* [14, 15] and endeavour to determine the statistical properties concerning the interaction of the *zpf* with the lattice sites in the system. In our model, the individual lattice sites interact with each other in a stochastic manner. The effect induced by all the lattice sites on a particular site is approximated by a single averaged effect. Let us consider that

$$\Phi = \langle \phi_s \rangle \quad (10)$$

gives the mean value of the state-variable ϕ_s and we define the fluctuation

$$\delta\phi_s = \phi_s - \Phi \quad (11)$$

Now, it is conventional to consider the following decomposition of the classical Hamiltonian

$$H = H_0 + H_1$$

such that the statistical features of H_0 can be calculated easily and exactly, whereas H_1 is comprised of all the parts of the Hamiltonian which are difficult to calculate. In our case, we consider the aforesaid decomposition such that

$$H_0 = - \sum_{\langle s_1, s_2 \rangle} E \phi_{s_1} \phi_{s_2} \quad (12)$$

and

$$H_1 = - \sum_s F \phi_s \quad (13)$$

where, we have assumed that the Hamiltonian corresponding to the interaction of the *zpf* with all sites $s \in S$ as the difficult part to calculate. Therefore, the partition functions for

H and H_0 are respectively given as

$$Z = \sum_s \exp(-\beta H)$$

and

$$Z_0 = \sum_s \exp(-\beta H_0)$$

Here, Z_0 is easy to calculate but not Z . Then, it is known that by means of the *Jensen's inequality* we have the relation

$$\langle \exp(-\beta H_1) \rangle_0 \geq \exp(-\beta \langle H_1 \rangle_0) \quad (14)$$

where the expectation value is considered with respect to the probability distribution generated by H_0 and

$$\frac{Z}{Z_0} = \langle \exp(-\beta H_1) \rangle_0$$

Now, this is known to lead to the *Bogoliubov inequality*

$$G \leq G_0 + \langle H_1 \rangle_0$$

where, $G = -k_B T \ln Z$ and $G_0 = -k_B T \ln Z_0$ are the respective free energies. But, we shall resort to equation (14) for our calculations. Particularly, considering only one lattice site (s) we have

$$\langle \exp\{-\beta F(\phi_s)\} \rangle \geq \exp(-\beta \langle H_1 \rangle) \quad (15)$$

Using relation (11) we obtain

$$\langle \exp(-\beta F\Phi - \beta F\delta\phi_s) \rangle \geq \exp(-\beta \langle H_1 \rangle_0) \quad (16)$$

Now, the left hand side can be looked upon as a *log-normal distribution*. We write

$$V = \exp(-\beta F\Phi - \beta F\delta\phi_s) = \exp(\mu + \sigma x)$$

where,

$$\mu = -\beta F\Phi$$

is the mean and

$$\sigma = -\frac{\beta F \delta \phi_s}{x}$$

is the standard deviation of the natural logarithm of the variable V and x is a *standard normal variable*. Now, the expectation value of V will be given by

$$\langle V \rangle = \langle \exp(-\beta F \Phi - \beta F \delta \phi_s) \rangle = \exp\{-\beta F \Phi + \frac{1}{2}(\frac{\beta F \delta \phi_s}{x})^2\} \quad (17)$$

Thus, from equation (16) we derive

$$\exp\{-\beta F \Phi + \frac{1}{2}(\frac{\beta F \delta \phi_s}{x})^2\} \geq \exp(-\beta \langle F \phi_s \rangle) \quad (18)$$

which after some calculations gives us finally

$$F \phi_s - \frac{\beta}{2}(\frac{F \delta \phi_s}{x})^2 \geq \langle F \phi_s \rangle \quad (19)$$

for a single lattice site s . Taking the sum for all lattice sites we will obtain the inequality for the entire lattice structure. Now, particularly if $\delta \phi_s \approx 0$ in the absence of fluctuations of the state ϕ_s , then from equation (19) we have the condition

$$F \phi_s \geq \langle F \phi_s \rangle \quad (20)$$

for a single lattice site $s \in S$. This means that in the absence of fluctuations the corresponding energy for a single lattice site must be greater or equal to the average value of all corresponding energy configurations for that site. Approximating the inequality (19) to be almost an equality we have the *standard normal variable*

$$x \approx -\sqrt{\beta} \frac{F \delta \phi_s}{\sqrt{F \phi_s - \langle F \phi_s \rangle}} \quad (21)$$

taking the negative value since the standard deviation has to be positive. Also, using the previously defined relation for the standard deviation, we have

$$\sigma \approx \frac{\sqrt{\beta}}{\sqrt{F \phi_s - \langle F \phi_s \rangle}} \quad (22)$$

where, the expectation values are taken with respect to the probability distribution generated by H_0 , the easier part of the Hamiltonian given by (12). Again, in terms of an *effective mean field* the Hamiltonian could have been written as

$$H = \frac{E \Phi^2 N z}{2} - (F + \Phi E z) \sum_s \phi_s$$

where, N is the number of lattice sites in the entire lattice structure and z is the *coordination number*. In that case, the equations (19) and (20) for a single lattice site would be respectively given as

$$G\phi_s - \frac{\beta}{2} \left(\frac{G\delta\phi_s}{x} \right)^2 \geq < G\phi_s > \quad (23)$$

and

$$G\phi_s \geq < G\phi_s > \quad (24)$$

where, $G = (F + \Phi Ez)$ is the *effective mean field* comprised of contributions from the interaction of neighbour-sites and the interaction of the *zpf* with the corresponding lattice site.

4 The Prevalent scale

In analogy to the author Sidharth's previous works [16, 17] we may write the state of the whole lattice structure as

$$\psi = \sum_s F_z \phi_p \quad (25)$$

where, F_z is the *zero point field* and ψ gives the state of the lattice structure considering the lattice sites as coherent Planck Oscillators and ϕ_p is the eigenstate (with some energy eigenvalues) corresponding to the *Planck state* for the lattice site $s \in S$. Now, in analogy with Sidharth's work

$$\psi = \sum_s F_{eff} \phi_c \quad (26)$$

where, F_{eff} is the effective field and ϕ_c is the *Compton state*. Specifically, this is the realistic case arising due to the interaction of the *zpf* with the Planck Oscillators.

Now, let us consider the lattice system in the above section to be in a mixed state of the statistical ensemble of the quantum states ϕ_c and ϕ_p . If the probabilities of finding the entire lattice system in any of the states ϕ_c, ϕ_p are given by P_c, P_p respectively then we have the *density operator* as

$$\hat{\rho} = \sum_{i=c,p} P_i |\phi_i\rangle \langle \phi_i| \quad (27)$$

where, $\sum_i P_i = 1$. Now, choosing an orthonormal basis $[\psi_k]$ we have the corresponding *density matrix* as

$$\rho_{kl} = \langle \psi_k | \hat{\rho} | \psi_l \rangle \quad (28)$$

Therefore the expectation value for any observable \hat{O} is given by

$$\langle \hat{O} \rangle = \sum_{i=c,p} P_i \langle \phi_i | \hat{O} | \phi_i \rangle = \sum_{kl} \langle \psi_k | \hat{\rho} | \psi_l \rangle \langle \psi_l | \hat{O} | \psi_k \rangle \quad (29)$$

Now, the expectation value of \hat{O} for each of the pure states ϕ_i is weighted by the probabilities P_i and since P_p is very small from our considerations in the first section, we observe that

$$\langle \hat{O} \rangle \approx P_c \langle \phi_c | \hat{O} | \phi_c \rangle \quad (30)$$

i.e., the expectation value of a physical observable will correspond to the pure state ϕ_c and be given by equation (30). This is to say that the *Compton scale* plays the rudimentary role in all phenomena of the quantum domain.

5 Applications

We now consider some interesting applications.

1) In analogy with the ferromagnetic case, we have been able to substantiate that the 2D universe undergoes a phase transition from the *Planck phase* to the *Compton phase*. This result should also hold in the case of a 3D universe. Now, if we take into consideration the author Sidharth's previous works [8, 9] then we have the *coherence parameter*

$$\xi = \frac{h}{mc} = \frac{2\pi\hbar}{mc}$$

where, $l_c = \frac{h}{mc}$ is the Compton length of a particle of mass ' m '. This Compton length is the fundamental aspect of the *Compton phase* or according to other authors [10, 11] the *Electroweak phase* which is the current phase of the universe.

Besides, in the light of the current paper it can be stated that if the *Compton scale* or *Compton phase* is taken as the fundamental aspect of the current stage of the universe, several phenomena can be elucidated very elegantly. The author Sidharth had shown that the Compton scale gives the description of an accelerating universe with a small non-zero cosmological constant [7]. Besides, the *GZK cutoff* [18] and certain phenomena in the atomic and subatomic scale, viz. the *Schwinger's correction terms* for the electron's gyromagnetic ratio [19], the *Lamb shift* [20] were shown to be explicable with subtlety resorting to the Compton length (corresponding to the Compton scale) of the electron. Without much ado, we can simply infer that the *Compton scale* plays as an underpinning central role in the understanding of various physical phenomena.

2) Now, let us consider the Dirac equation

$$(\gamma^\mu p_\mu - m)\psi = 0$$

where, γ^μ are 4×4 matrices obeying the Clifford algebra and ψ is a 4-component spinor given by

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}$$

ϕ and χ being 2-component spinors. Traditionally the spinors ϕ and χ have been called the low energy and high energy solutions and as Feshbach and Villars showed [21] they form two separate coupled equations. In this particle interpretation following Weisskopf, they call ϕ the positive energy or particle solutions and χ as the negative energy or antiparticle solutions. The author Sidharth demonstrated at length [22, 23, 24] using the fact that outside the Compton wavelength ϕ dominates, while near the Compton wavelength it is χ that dominates [25]. This is equivalent to a particle-antiparticle combination on the lines of isospin. This would explain several phenomena such as the appearance of antiparticles at high energy and particle at lower energy. In this sense the four component Dirac spinor resembles the 2 state particle-antiparticle system. Now, if we consider the model presented in this paper and think of the particles and antiparticles as two different states then it would explain why the particles dominate over the antiparticles outside the Compton length. The presence of an electromagnetic field triggers such phase transitions and the number of particles are greater than that of antiparticles, thereby making the positive energy component dominant.

Again, below the Compton length the field configuration is such that the number of antiparticles is greater than the particles, thereby making the negative energy component dominant. In this manner, resorting to our model one can elegantly elucidate on the dominance of the particles over the antiparticles above the Compton length.

3) Also, we would like to stress about an interesting idea in terms of the results of the current paper. The author Sidharth [26] had proposed a link between *chirality* and the *cosmic origins of life*. This *chirality* carries over to molecules and such a molecule displaying handedness is called an *enantiomer*. Amino acids are found to be *enantiomers*. Observations point to the fact that there are equal number of left-handed and right-handed amino acids (a key ingredient of living organisms). Such a mixture of equal amount of two *enantiomers* is termed as *racemic*. But, an interesting fact is that *racemic* mixtures are optically inactive. On the contrary, separate *enantiomer* constituents are optically active. Thus, in reality the amount of chiral *enantiomers* are slightly greater.

Now, suppose we apply our model to the case of human body. Let us assume that the human body possesses only right-handed amino acids and that the molecules form a lattice-like structure. Since, there is an extremely faint electromagnetic field (due to presence of various ions in the body) one can infer from the essence of this paper that there is a phase transition of the right-handedness to the left-handedness of the amino acids, by dint of the said field. Due to the phase transition, the probability of the amino acids taking up a chiral configuration is greater than that of the right-handed configuration. This would

substantiate why there is a surplus in the amount of chiral *enantiomers* (amino acids).

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